

# Technical Notes

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## Improved Method for the Measurement of Turbulence Quantities

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### Nomenclature

$D$	= diameter of jet
$E_L$	= total voltage output of hot wire for linearized operation
$e$	= fluctuating voltage of hot wire
$G, K$	= directional sensitivity coefficients of the hot wire
$S$	= calibration factor of the hot wire for linearized operation, $V \cdot s/m$
$U, V, W$	= mean velocities in the $X, Y$ , and $Z$ directions, respectively
$U_0$	= mean velocity at the exit of the nozzle
$U_m$	= mean velocity at the centerline of the jet
$u, v, w$	= fluctuating components of the velocities in the $X, Y$ , and $Z$ directions, respectively
$\alpha$	= angle between the plane in which the wire lies and the reference plane
$\beta$	= inclination of the hot wire

### Introduction

THE basic problem in hot-wire anemometry is to relate the velocity components to the measured hot-wire voltages. The conventional methods as discussed by Hinze<sup>1</sup> and Champagne and Sleicher<sup>2</sup> make use of the time-averaged form of the hot-wire response equation. As it is not possible to solve the square root expression for the velocity components directly by time averaging, the expression within the square root is first expanded into a series and the third- and higher-order correlations are assumed to be negligible and the expressions for the velocity components are derived. In highly turbulent flows these higher-order correlations are not negligible, and the series expansion is not permissible in extreme cases. Use of three-sensor probes with digital data analysis will enable the determination of the instantaneous velocity components without approximations. But the method is very expensive and three closely placed sensors are likely to interfere with each other.

Since all of the difficulty is caused by a square root, a possible solution is to use the square of the response equation before time averaging. This corresponds to evaluating the squared signal  $E^2$  rather than  $\bar{E}$ . In the present investigations a general three-dimensional flow is considered and exact

formulas for the quantities  $\overline{U^2 + u^2}$ ,  $\overline{V^2 + v^2}$ ,  $\overline{W^2 + w^2}$ ,  $\overline{UV + uv}$ ,  $\overline{VW + vw}$ , and  $\overline{UW + uw}$  are determined. After determining the mean velocity components  $U, V$ , and  $W$  by a suitable method, the turbulence quantities  $\overline{u^2}$ ,  $\overline{v^2}$ ,  $\overline{w^2}$ ,  $\overline{uv}$ ,  $\overline{vw}$ , and  $\overline{uw}$  can be determined.

### Analysis

Consider an inclined wire of angle  $\beta$  lying in the plane  $OX'Y'O'$  which makes an angle  $\alpha$  with the plane  $XOY$  as shown in Fig. 1. It can be shown that the effective cooling velocity is given by

$$U_{\text{eff}}^2 = [(U+u)\cos\alpha\sin\beta + (W+w)\sin\alpha\sin\beta - (V+v)\cos\beta]^2 + G^2[(U+u)\sin\alpha - (W+w)\cos\alpha]^2 + K^2[(U+u)\cos\alpha\cos\beta + (W+w)\sin\alpha\sin\beta + (V+v)\sin\beta]^2 \quad (1)$$

where  $G$  and  $K$  are the directional sensitivity coefficients of the wire. For a linearized system we have

$$E_L = SU_{\text{eff}}$$

where  $S$  is the sensitivity factor to be determined from the calibration of the wire.

$$E_L^2 = S^2 U_{\text{eff}}^2$$

Time averaging,

$$\overline{E_L^2} = \overline{E_L^2} + \overline{e^2} = S^2 \overline{U_{\text{eff}}^2} \quad (2)$$

By time averaging Eq. (1) and substituting in Eq. (2), we have

$$\begin{aligned} (\overline{E_L^2} + \overline{e^2})/S^2 = & (\overline{U^2 + u^2})[\cos^2\alpha\sin^2\beta + G^2\sin^2\alpha \\ & + K^2\cos^2\alpha\cos^2\beta] + (\overline{V^2 + v^2})(\cos^2\beta + K^2\sin^2\beta) \\ & + (\overline{W^2 + w^2})[\sin^2\alpha\sin^2\beta + G^2\cos^2\alpha + K^2\sin^2\alpha\cos^2\beta] \\ & + (\overline{UV + uv})\sin 2\beta\cos\alpha(K^2 - 1) \\ & + (\overline{VW + vw})\sin 2\beta\sin\alpha(K^2 - 1) \\ & + (\overline{UW + uw})\sin 2\alpha(\sin^2\beta - G^2 + K^2\cos^2\beta) \end{aligned} \quad (3)$$

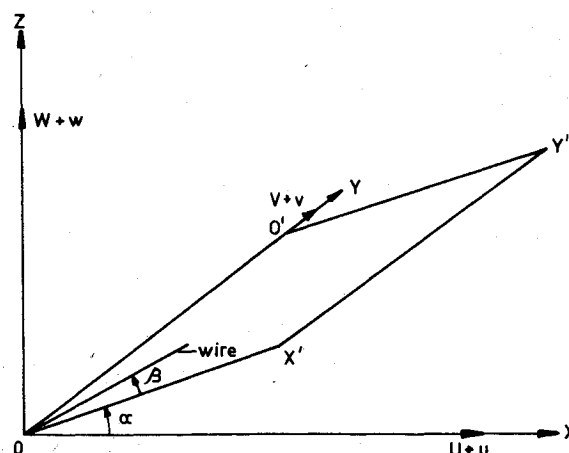


Fig. 1 Orientation of the hot wire.

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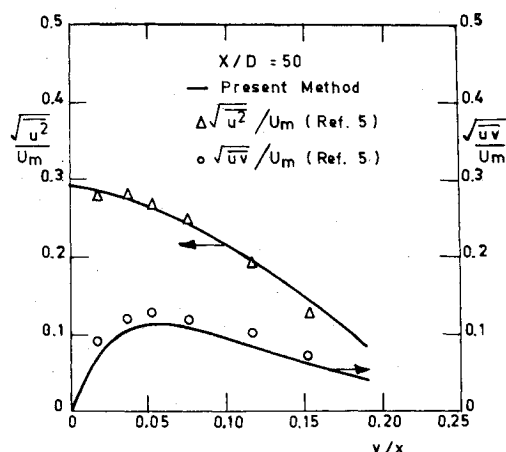


Fig. 2  $\overline{uv}/U_m^2$  distribution at  $X=50D$ .

If an  $X$  probe is used, for one wire  $\beta=45$  deg and for the other  $\beta=135$  deg. Bartenwerfer<sup>3</sup> has used an  $X$  probe and the exact formulas for  $UV+uv$ ,  $VW+vw$ , and  $UW+uw$  are determined from the orientations of the wire in the  $XY$ ,  $YZ$ , and  $XZ$  planes and formulas for  $U^2+\overline{u^2}$ ,  $V^2+\overline{v^2}$ , and  $W^2+\overline{w^2}$  are determined from three suitable orientations which are not symmetrical to any spatial direction. This requires the  $X$  probe to be introduced along two different axes.

In the present investigations, instead of an  $X$  probe, a new probe in which a straight wire is combined with a 45 deg wire has been used. With such a probe the necessity to measure along two different axes is eliminated. The new probe is fabricated by modifying a standard dual-sensor probe in which the two wires are parallel to each other. The probe is rotated about the axis  $OY$  (Fig. 1). The linearized voltage and the rms voltage of the 45 deg wire are measured at angles  $\alpha=0, 45, 90, 180$ , and  $270$  deg. The response equations of the 45 deg wire can be obtained by substituting the values of  $\alpha$  in Eq. (3) with  $\beta=45$  deg. The linearized voltage and the rms voltage of the straight wire are measured at  $\alpha=90$  deg. The response equation for the straight wire in this position can be obtained from Eq. (3) with  $\alpha=90$  deg and  $\beta=0$ . The five response equations of the 45 deg wire and the response equation of the straight wire are solved to determine the quantities  $\overline{U^2}+\overline{u^2}$ ,  $\overline{V^2}+\overline{v^2}$ ,  $\overline{W^2}+\overline{w^2}$ ,  $UV+uv$ ,  $VW+vw$ , and  $UW+uw$ . The turbulence quantities can be determined if the mean velocities are known. The direction and magnitudes of the mean velocities  $U$ ,  $V$ , and  $W$  can be determined using the 45 deg inclined wire itself following the method suggested by Moussa and Eskinazi.<sup>4</sup> But it requires extensive calibration of the inclined wire to determine both the direction and magnitudes of the mean velocities. An alternate method is to use three-dimensional pitot probes (three hole or five hole) to determine the mean velocities. With the availability of micromanometers capable of measuring very small pressure differentials accurately, the determination of the mean velocities to the required degree of accuracy should not pose any serious problem.

As a test case the method has been applied to the determination of turbulence intensities at  $X=50D$  and  $60D$  in a free jet and the results are compared with the results of Wygnanski and Fiedler<sup>5</sup> and Rodi.<sup>6</sup>

### Experimental Setup

The jet emerged from a nozzle 10 mm in diameter at a velocity of 80 m/s and the variation of the jet velocity was within 1%. The temperature variation was within 2°C. Constant temperature anemometers and linearizers were used. A time constant of 100 s was used and the voltages were taken after a settling time of 10 min. The mean velocity in the axial direction  $U$  was measured with the straight wire itself and the crosswise mean velocities  $V$  and  $W$  were assumed to be negligible.

### Results and Discussion

The variation of the mean velocity  $U$  at the center of the jet with distance  $X$  is in good agreement with the measurements of Wygnanski and Fiedler.<sup>5</sup> The hypothetical origin of the jet is located at about 7 diam in front of the nozzle. The mean velocity profiles across the jet at  $X=50D$  and  $60D$  are also in good agreement with those of Wygnanski and Fiedler<sup>5</sup> and Rodi.<sup>6</sup> The distributions of  $\sqrt{\overline{u^2}}/U_m$ ,  $\sqrt{\overline{v^2}}/U_m$ , and  $\sqrt{\overline{w^2}}/U_m$  across the jet at  $X=50D$  and  $60D$  are also in good agreement with the measurements of Wygnanski and Fiedler<sup>5</sup> and Rodi<sup>6</sup> at points within  $y/x=0.08$ ; however, beyond  $y=0.08X$  (i.e., in the outer region of the jet) some difference is observed. The maximum deviation in respect to the above three variables is about 6%. These results are not presented due to space limitations. Figure 2 shows the distribution of  $\sqrt{\overline{u^2}}/U_m$  and  $\sqrt{\overline{uv}}/U_m$  across the jet at  $X=50D$ . It can be seen that the distribution at  $\sqrt{\overline{u^2}}/U_m$  agrees well with the measurements of Wygnanski and Fiedler.<sup>5</sup> As far as  $\sqrt{\overline{uv}}/U_m$  is concerned, while the result follows the same trend as those of Wygnanski and Fiedler, quantitatively there is some difference. The observed differences might be due to

1) Wygnanski and Fiedler have eliminated all room draughts, which tend to alter the flow, by enclosing the entire jet in a case. In the present work, the experiments were usually conducted at times when there was no disturbance; however, the jet was not enclosed completely. This might have resulted in some inaccuracies in the measurements.

2) From Fig. 2, it can be seen that the turbulent shear stress is nearly half the normal stress in magnitude at all points. Hence, even small errors in the measurement of turbulent quantities will be reflected to a large extent in the values of shear stress. The observed difference can be considerably reduced by proper signal conditioning and by using larger integrator time constants. However, on the whole the agreement is quite satisfactory.

The present method has the following advantages:

- 1) The formulas used in the method are exact, as no approximations have been made.
- 2) The use of linearized operation enables the method to be used for flows of high turbulence intensities.
- 3) The method does not require orienting the probe along the streamline at the point of measurement.

The limitations of the method are:

- 1) The mean velocities have to be determined accurately by suitable means. This can be done either by using the inclined wire itself or by using well-calibrated three-dimensional pitot probes.
- 2) The interference between the two wires may lead to some inaccuracies. This can be avoided by using two separate probes: one for the straight wire and another one for the inclined wire. But this will increase the time of measurement and the improvement in accuracy is not very great.

### References

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